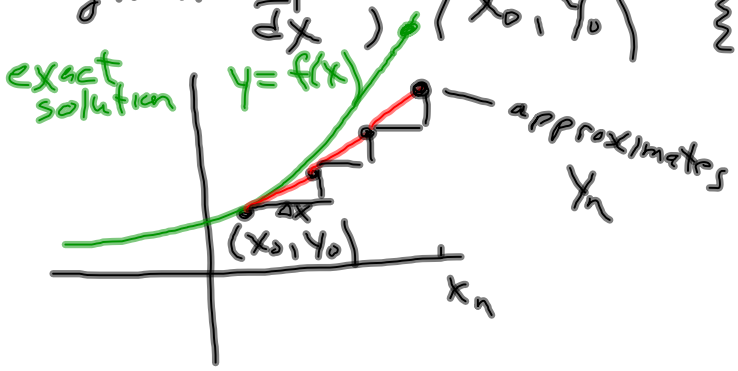
6.1 Euler's method

Numerical method to solve differential equations (approximate)

given $\frac{dy}{dx}$, (x_0, y_0) {initial value}

find $y_n = f(x_n)$ {final value}



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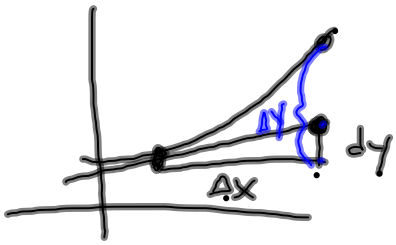
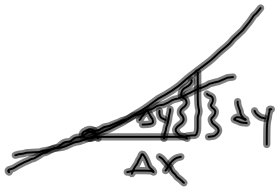


differential der differential

$$dy = \frac{dy}{dx}\,dx = \frac{dy}{dx}\cdot \Delta x$$



$$y_{new} = y_{old} + \Delta y$$

$$\approx y_{old} + \frac{dy}{dx}\cdot dx$$

$$y_{new} = y_{old} + f'(x,y)\cdot \Delta x$$

$$\boxed{y_{n+1} = y_n + f'(x,y)\,\Delta x}$$

Dec 8-1:26 PM

$\frac{dy}{dx} = x+y$

$f'(x,y) = x+y$

$f(2) = 0$ $\quad f(3) = ?$

$x = 2$ $\quad x = 3$ given

$y = 0$ $\quad y = ?$ $\quad \Delta x = .2$

≈ 3.46496

x	y	y'
2	0	2
2.2	.4	2.6
2.4	.92	3.32
2.6	1.584	
2.8	2.4208	
3	3.46496	

new old

$y_1 = 0 + 2(.2) = .4$

$y_2 = .4 + 2.6(.2) = .92$

$y_3 = .92 + 3.32(.2) =$

$y_{n+1} = y_n + f' \cdot \Delta x$

Dec 8-1:33 PM

$\frac{dy}{dx} = 2x - y$

$f(2) = 3$ $\quad f(1.5) = ?$

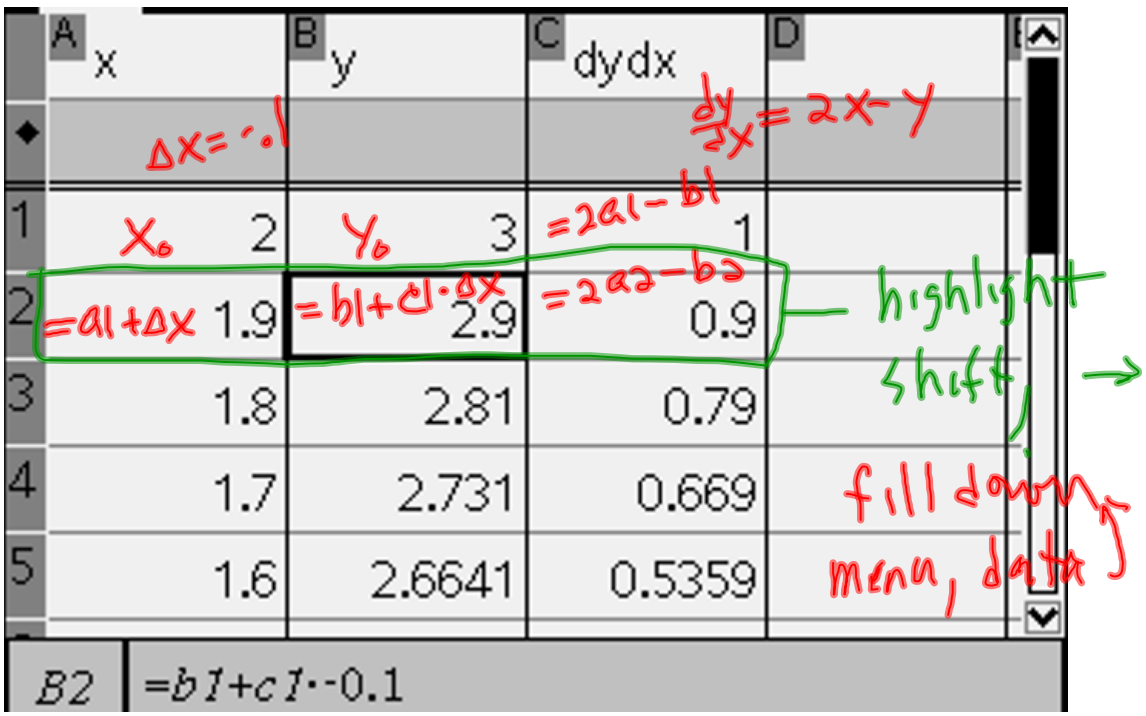
$x = 2$

$y = 3$ $\quad \Delta x = -.1$

$y_1 = 3 + 1 \cdot (-.1) = 2.9$

x	y	y'
2	3	1
1.9	2.9	.9
1.8		
1.7		
1.6		
1.5	2.61051	

Dec 8-1:46 PM


A x
B y
C dydx
D
Δx=.1
dy/dx = 2x-y
1 2 3 1
X0 Y0 =2a1-b1
2 =a1+Δx 1.9 =b1+c1·Δx 2.9 =2a2-b2 0.9
highlight
shift →
3 1.8 2.81 0.79
4 1.7 2.731 0.669
fill down
5 1.6 2.6641 0.5359
menu, data
B2 =b1+c1·-0.1

Dec 8-2:07 PM